M.Sc. 2nd Semester Examination, 2023
Department of Mathematics,
Mugberia Gangadhar Mahavidyalaya
(Fluid Mechanics)
Paper MTM – 201
FULL MARKS: 50 : : Time : 02 hours

(***Candidates are required to give their answers in their own words as far as practicable***)

1. Answer any four questions **2×4=8**

(a) Suppose a viscous fluid (of density 900Kg/m^3 and viscosity 0.5Ns/m^2) is flowing through a pipe of radius 20mm at a speed 3m/s. Calculate the Reynolds number for this flow. **2**

(b) Find the substantial derivative of the steady state velocity field represented by the velocity vector $\vec{V}$= (-3x, -3y, 6z). **2**

(c) What is Newtonian and Non-Newtonian fluids? Discuss with example. **2**

(d) Write the assumption of the boundary layer theory. **2**

(e) What are the differences between laminar and turbulent flows? **2**

(f) The diameter of a pipe at the section 1 and 2 are 10cm and 20cm respectively. If the velocity of water through the pipe at section 1 is 7 m/s. determine also the velocity at section 2. **2**

2. Answer any four questions **4×4=16**

(a) Draw an infinitesimally small moving element and show all the surface forces acting along the y-direction on the element. Finally find the net surface and body forces acting on that element. **4**

(b) Discuss the model of an infinitesimally small element fixed in space. **4**

(c) Determine the equation of the rate of work done on element due to body and free surface. **4**

(d) Describe the $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial t^2}$ $using$ Crank-Nicolson scheme and hence write the algebraic expression in a matrix form for the case of Neumann boundary conditions. **4**

(e) Make the two-dimensional unsteady energy equation (without radiation and viscous dissipation terms) in non-dimensional form (in terms of Reynolds number $Re = \frac{UL}{v}$ and Prandtl number $Pr = \frac{v}{\alpha}$) with the help of characteristics time, length and velocity as L/U, L and U, respectively, with non-dimensional temperature $T = \frac{T^*}{T_0}$ where T_0 is the reference temperature and symbols have their usual meaning. **4**

(f) An incompressible velocity fields is given by $u = a(x^2 - y^2)$, $v = -2axy$ and ω= 0. Determine under what conditions it is a solution to the NavierStokes momentum equation for the case of without any body forces. Assuming that these conditions are met, determine the resulting pressure distribution. **4**

3. Answer any two questions **8×2=16**

(a) Consider steady, laminar, fully developed flow between two parallel plates separated by a distance $2H$. The fluid is driven between the plates by an applied pressure gradient in the x-direction. It is assumed that conduction in the y-direction is much greater than conduction in x-direction. **4+4**

(i) Determine the fully developed velocity distribution of the fluid as a function of the mean velocity.

(ii) Determine the fully developed temperature distribution as a function of the surface and mean temperatures.

(b) (i) Write An incompressible velocity fields is given by u is unknown, $v = c(x^2 - 2xy + y^2), w = a - y^2$. What must the form of the velocity component u be? **2+6**

(ii) Show Derive the energy equation for non-Newtonian, incompressible, viscous fluid flow with negligible radiation effects.

(c) (i) Write all the equations of motion in terms of eddy viscosities. **2+6**

(ii) For the ocean with horizontal and vertical length scales 1000KM and 2KM, respectively and horizontal speed of order 0.15m/s, scale all the above equations written in part-(a) and reduces to approximated equations with order of accuracy 1%.

(d) A circular cylinder is moving in a liquid at rest to infinite to calculate the forces on the cylinder owing to the pressure of the field. **8**

[Internal Assesment-10 marks]

M.Sc. 2nd Semester Examination, 2023
Department of Mathematics,
Mugberia Gangadhar Mahavidyalaya
(Numerical Analysis)
Paper MTM – 202
FULL MARKS: 50 : : Time : 02 hours

(***Candidates are required to give their answers in their own words as far as practicable***)

1. Answer any four questions **2×4=8**

(a) Write the merits and demerits of the LU-decomposition method to solve a system of linear equations. **2**

(b) Is it possible to write any polynomial in terms of the Tchebyshev polynomial? Explain with an example. **2**

(c) Suppose the maximum (in magnitude) eigenvalue of a matrix is known. How can you calculate the minimum (in magnitude) eigenvalue of the same matrix? **2**

(d) What are the advantages to approximate a function using orthogonal polynomials? **2**

(e) Let $S(x)=14T_4(x)+7T_3(x)+2T_2(x)-23T_0(x)$. Find the value of S(1). **2**

(f) Let $a, b, c \in \mathbb{R}$ be such that the quadrature rule **2**

$$\int_{-1}^{1} f(x)dx = af(-1) + bf'(0) + cf'(1)$$

is exact for all polynomials of degree less than or equal to 2. Then find the value of $(a+b+c)$.

2. Answer any four questions **4×4=16**

(a) Explain a finite difference method to solve the wave equation **4**

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad t > 0, \quad 0 < x < 1$$

where initial conditions $u(x,0) = f(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{(x,0)} = g(x)$, $0 < x < 1$ and boundary conditions $u(0,t) = \varphi(t)$ and $u(1,t) = \psi(t)$, $t \geq 0$.

(b) Solve the following system of equations $x = (8x - 4x^2 + y^2 + 1)/8$ and $y = (2x - x^2 + 4y - y^2 + 3)/4$ starting with $(x_0, y_0) = (1.1, 2.0)$, using Seidel iteration method. **4**

(c) Use the 3-point Gauss-Tchebyshev quadrature method to find the value of $\int_{-1}^{1} (\sec x + 2x^4)\, dx$. **4**

(d) Let $f(x) = \begin{cases} 2x^3 - 4.5x^2 + 3x + 5, & 0 \le x \le 1 \\ -x^3 + 4.5x^2 - 6x + 8, & 1 \le x \le 2. \end{cases}$ **4**

Show that $f(x)$ is a cubic spline.

(e) Given $\frac{dy}{dx} = x - y^2$ with $x = 0, y = 1$. Find $y(0.2)$ by second and fourth order Runge-Kutta methods. **4**

(f) Describe 3-point Gauss – Legendre quadrature formula. **4**

3. Answer any two questions **8×2=16**

(a) (i) What is the residual? **2+6**

(ii) Let $(2, 2)$, $(-1.5, -2)$, $(4, 4.5)$ *and* $(-2.5, -3)$ be a sample. Use least squares method to fit the line $y = a + bx$ based on this sample and estimate the total error.

(b) Explain the Baristow method to find all roots of a polynomial equation. **8**

(c) Explain the successive overrelaxation method to solve a system of linear equations. **8**

(d) Describe Jacobi's method to find all eigenvalues and eigenvectors of a symmetric matrix. **8**

[Internal Assesment-10 marks]

M.Sc. 2nd Semester Examination, 2023
Department of Mathematics,
Mugberia Gangadhar Mahavidyalaya
(Abstract and Linear Algebra)
Paper: MTM – 203

FULL MARKS: 50 : : Time : 02 hours

(***Candidates are required to give their answers in their own words as far as practicable***)

Unit I: **Abstract Algebra (FULL MARKS: 25)**

1. Answer any two questions **2×2=4**

(a) How many sylow-2 subgroups are there for any group of order 6? explain. **2**

(b) Determine the number of elements of order 5 in $Z_{25} \oplus Z_5$. **2**

(c) Determine the degree of $[\mathbb{Q}(\sqrt{3} + 2\sqrt{2}) : \mathbb{Q}]$. **2**

(d) Prove that if $[F(\alpha) : F]$ is odd then $F(\alpha) = F(\alpha^2)$. **2**

2. Answer any two questions **4×2=8**

(a) Show that it is impossible to construct a regular 9-gon by ruler and compass. **4**

(b) Let $K \subseteq F$ be a field extension and $\alpha \in F$ be algebraic over K. Then show that there exists a unique monic irreducible polynomial $f(x) \in K[x]$ such that $f(\alpha) = 0$. **4**

(c) Derive the class equation of quaternion group Q_8. **4**

(d) (e) State and prove Sylow's 2nd theorem. **4**

3. Answer any one question **8×1=8**

(a) (i) Show Let E be a field and G a finite group of automorphisms of E. Then show that E/E^G is a finite Galois extension. **5+3**

(ii) Show that the Galois group of the Galois extension $\mathbb{F}_{q^n}/\mathbb{F}_q$ is a cyclic group of order n.

(b) (i) Show that any group of order 12 is isomorphic to S={x,y : x^4 = 1, y^3= 1, yx=y^2x}. **4+2+2**

(ii) Show that $\frac{4\mathbb{Z}}{12\mathbb{Z}} \cong \mathbb{Z}_3$

(iii) Define splitting field with example.

[Internal Assesment-05 marks]

Unit II: **Linear Algebra (FULL MARKS: 25)**

1. Answer any two questions **2×2=4**

(a) Justify the statements as true or false **2**

(i) Every linear operator has an adjoint.

(ii) The adjoint of a linear operator is unique.

(b) What do you mean by a normal operator? Are all self-adjoint operators normal? Justify. **2**

(c) Define Bilinear form on vector space V over a field F. **2**

(d) Give an example a complex symmetric matrices need not be normal. **2**

2. Answer any two questions **4×2=8**

(a) Let V be the vector space of all polynomial functions p from R into R which have degree 2 or less. Define three functions on V given by, $f_1(p) = \int_0^1 p(x)dx,$ $f_2(p) = \int_0^2 p(x)dx,$ $f_3(p) = \int_0^{-1} p(x)dx.$ **4**

Show that $\{f_1, f_2, f_3\}$ is a basis of V^* (dual basis). Determine a basis for V such that $\{f_1, f_2, f_3\}$ is its dual basis.

(b) Let V be an inner product space, and let T be a linear operator on V. Then T is an orthogonal projection if and only if T has an adjoint $T^*$and $T^2 = T = T.^*$ **4**

(c) Find all possible Jordan canonical forms for a linear operator T: V to V (vector space) whose characteristic polynomial is $(t - 2)^3(t - 5)^5$. In each case, find the minimal polynomial $m(t)$. **4**

(d) Let v be a finite dimensional vector space , and define $\psi: V \to V^{**}$ by $\psi(x) = \bar{x}$ then prove that ψ is an isomorphism. **4**

3. Answer any one question **8×1=8**

(a) (i) State and prove Sylvester's Law of Inertia on finite dimensional real vector space V. **6+2**

(ii) Prove that the sum of two bilinear forms is a bilinear form.

(b) (i) Let T and U be self-adjoint operators on an inner product space V. Prove that TU is self-adjoint if and only if $TU = UT$. **3+3+2**

(ii) Let T be a normal operator on a finite-dimensional real inner product space V whose characteristic polynomial splits. Prove that V has an orthonormal basis of eigenvectors of T. Hence prove that T is self-adjoint.

(iii) If T is normal and $T^3 = T^2$, show that T is idempotent. If normality of T is dropped, does the conclusion still true?

[Internal Assesment-05 marks]

M.Sc.2nd Semester Examination, 2023
Department of Mathematics,
Mugberia Gangadhar Mahavidyalaya
(General Theory of Continuum Mechanics)
Paper MTM – 205
FULL MARKS: 50 : : Time : 02 hours

(***Candidates are required to give their answers in their own words as far as practicable***)

1. Answer any four questions **2×4=8**

(a) Find the constitutive equation of perfect fluid. **2**

(b) Define stress quadric. **2**

(c) Prove that all principal strains are real. **2**

(d) If the deformation of a body is defined by the displacement components $u_1 = k(3{X_1}^2 + X_2), u_2 = k({X_2}^2 + X_3), u_1 = k(X_3 + X_1)$, where $k > 0$. Compute the extension of a line element that passes through the point (1, 1, 1) in the direction ($\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$). **2**

(e) Find the complex potential due to a source. **2**

(f) The velocity (u, v, w) of a fluid at a point $P(x, y, z)$ is given by $= \frac{-2xyz}{x^2+y^2}$, $v = \frac{yz}{x^2+y^2}$ and $w = \frac{z}{x^2+y^2}$. Find the rate at which density of the fluid at point P is decreasing in the flow field. **2**

2. Answer any four questions **4×4=16**

(a) Derive the Helmholtz's equations of vorticity for perfect fluid. **4**

(b) Prove that, in irrotational motion of an incompressible fluid, the maximum value of the speed occur on the boundary of the fluid. **4**

(c) Prove that volumetric strain or cubical dilatation is equal to the sum of three linear strains. **4**

(d) The stress matrix at a point $P(x_i)$ in a material is given by **4**

$$(T_{ij})=\begin{pmatrix} x_1x_3 & x_3^2 & 0 \\ x_3^2 & 0 & -x_2 \\ 0 & -x_2 & 0 \end{pmatrix}$$

Find the stress vector at the point $Q(1,0,-1)$ on the surface $x_1 = x_2^2 + x_3^2$.

(e) State and prove Kelvin's Circulation Theorem. **4**

(f) Show that $\frac{x^2}{a^2}\varphi(t) + \frac{y^2}{b^2}\frac{1}{\varphi(t)} = 1$ is a possible form of boundary surface of a liquid. **4**

3. Answer any two questions **8×2=16**

(a) Define the rigid body deformation. Derive the Lagrangian finite strain tensor $(\boldsymbol{r_{ij}})$ and change in the angle between two line elements. Hence show that the body has undergone only rigid body deformation if $\boldsymbol{r_{ij}} = \mathbf{0}$. **1+6+1**

(b) An infinite mass of fluid acted on by a force $\boldsymbol{\mu r}^{-(3/2)}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere $\boldsymbol{r = c}$ in it, show that the cavity will be filled up after an interval of time $\sqrt{(\mathbf{2/5}\boldsymbol{\mu})}\ \mathbf{c}^{5/4}$. **8**

(c) The stress tensor at a point is given by **2+4+2**

$$(T_{ij}) = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Determine the principal stresses and corresponding principal directions. Also check on the invariance of θ_1, θ_2 and θ_3.

(d) Derive the basic elastic constants for isotropic elastic solid. **8**

[Internal Assesment-10 marks]

M.Sc. 2nd Semester Examination, 2023
Department of Mathematics,
Mugberia Gangadhar Mahavidyalaya
(General Topology)
Paper MTM – 206
FULL MARKS: 25 : : Time : 01 hour

(*Candidates are required to give their answers in their own words as far as practicable*)

1. Answer any two questions **2×2=4**

(a) Show that k-th topology is finer than the standard topology. **2**

(b) Define quotient topology. **2**

(c) What do you mean by topological imbedding? Illustrate with an example. **2**

(d) Define locally compact space with an example. **2**

2. Answer any two questions **4×2=8**

(a) Let Y be an ordered set in the order topology. Let $f, g: X \to Y$ be continuous. Show that the set $\{x \in X | f(x) \leq g(x)\}$ is closed in X. **4**

(b) Show that $\mathbb{R}^w$ in the box topology is not connected. **4**

(c) Show that every path connected space is a connected space. Is converse also true? Explain. **4**

(d) Show that every compact Hausdorff space is normal space. **4**

3. Answer any one question **8×1=8**

(a) (i) Let X and Y be two topological space and $f: X \to Y$. Then the following are equivalent. **(2+2)** +

I. f is continuous.

II. for every closed set B of Y the set $f^{-1}(B)$ is closed in X.

(ii) Examine the compactness of the following sets over the interval $(0,1)$ **(2+2)**

I. $\left\{\left(sin^2\left(\frac{n\pi}{100}\right), cos^2\left(\frac{n\pi}{100}\right)\right): n \in \mathbb{N}\right\}$

II. $\left\{\left(\frac{1}{2}e^{-\pi}, 1 - \frac{1}{(n+1)^2}\right): n \in \mathbb{N}\right\}$.

(b) (i) Show that a subspace of a regular space is regular. **3+5**

(ii) Define Lindelof space. Give example to show that the product of two Lindelof space need not be Lindelof.

[Internal Assesment-05 marks]

M.Sc. 2nd Semester Examination, 2023
Department of Mathematics,
Mugberia Gangadhar Mahavidyalaya
(Language: C- Programming with Numerical Methods)
Paper MTM – 297
FULL MARKS: 25 : : Time : 02 hours

Group A

Answer one question which is selected by lottery: 10×1=10

1. Write a program in C to find the key number 25 from the list of numbers {15, 47, 78, 12, 56, 78, 25, 34, 45, 98} using Linear search technique.

2. Write a program in C to find the key number 25 from the list of sorted numbers {12, 15, 25, 34, 45, 47, 51, 56, 87, 98} using Binary search technique.

3. Write a program in C to sort the list of numbers {15, 47, 81, 12, 56, 78, 25, 34, 45, 98} using Bubble sort technique.

4. Write a program in C to sort the list of numbers {15, 47, 78, 12, 56, 88, 25, 34, 45, 98} using Insertion sort technique.

5. Write a program in C to sort the list of numbers {15, 47, 75, 12, 56, 78, 25, 34, 45, 98} using Selection sort technique.

6. Write a program in C to find the number of occurrences of a letter ‘a’ in a given string “Student stays focused on the task at hand”.

7. Write a program in C to check whether a given string is palindrome nature or not. Test it for the strings: “deleveled”, “redder”, “mathematics”.

8. Write a program in C to rewrite the name with surname first followed by initials of first and middle name. Test it for the names: (i) Sunil Kumar Dey (ii) Manas Kumar Mondal (iii) Soma Rani Majhi (iv) Sathi Jana

9. Write a program in C to display the string "The reverse is true as well" in a reverse order.

10. Write a program in C to search the string "quality" in the given string (Pattern Matching) "Student completes work with quality in mind".

11. Write a program in C to sort the names in alphabetic order. Test it for the names: (i) Sunil Kumar Dey (ii) Manas Kumar Mondal (iii) Soma Rani Majhi (iv) Sathi Jana (v) Rathin Samanta.

12. Write a program in C to find the letter 'd' and replace by the letter 'b' in a given string "Student is a self-motivated worker".

13. Write a program in C to find the word "daily" and replace by the word "weekly" in a given string "Student always completes daily assignments in a timely manner".

14. Write a program in C to print all combinations of letters of a word "MATH".

15. Write a program in C to convert the name into abbreviation form. Test it for the names: (i) Sunil Kumar Dey (ii) Manas Kumar Mondal (iii) Soma Rani Majhi (iv) Sathi Jana (v) Rathin Samanta.

Group B

Answer one question which is selected by lottery: 10×1=10

1. Write a program in C to evaluation of determinant by Gauss elimination method, using partial pivoting. Using this code compute the determinant of the following matrix

$$A=\begin{bmatrix} 2 & 0 & 4 \\ 4 & 6 & 1 \\ 5 & 1 & -2 \end{bmatrix}$$

2. Write a program in C to find matrix inverse by partial pivoting. Find the inverse of the following matrix $A=\begin{bmatrix} 2 & 4 & 5 \\ 1 & -1 & 2 \\ 3 & 4 & 5 \end{bmatrix}$

3. Write a program in C to find the roots of polynomial equation.

4. Write a program in C to solve the following system of equations by matrix inverse method

$$x + 2y + 3z = 10,\ x + 3y - 2z = 7,\ 2x - y + z = 5$$

5. Write a program in C to solve the equations by Gauss elimination method. $2x_1 + x_2 + x_3 = 4$, $x_1 - x_2 + 2x_3 = 2$, $2x_1 + 2x_2 - x_3 = 3$.

6. Write a program in C to solve the following system of equations by Gauss-Seidal's iteration method, correct up to four decimal places.

$$27x + 6y - z = 54,\ 6x + 15y + 2z = 72,\ x + y + 54z = 110.$$

7. Write a program in C to solve the following tri-diagonal system of equation.

$$x_1 + x_2 = 3,\ -x_1 + 2x_2 + x_3 = 6,\ 3x_2 + 2x_3 = 12.$$

8. Write a program in C to obtain a quadratic polynomial approximation to $f(x) = e^{-x}$ using Lagrange's interpolation method, taking three points $x = 0, 1/2, 1$.

9. The following table gives pressure of a steam plant at a given temperature. Using Newton's formula, write a program in C to compute the pressure for a temperature of 142°C.

Temperature °C :	140	150	160	170	180
Pressure, kgf/cm^2:	3.685	4.854	6.302	8.076	10.225.

10. The population of a town in decennial census were as under. Write a program in C to estimate the population for the year 1955.

Year:	1921	1931	1941	1951	1961
Population (in crore):	46	68	83	95	105.

11. Write a program in C to find the value of the integration of $\int_0^1 \frac{1}{1+x^2}$ dx by Monte Carlo method for different values of N.

12. Use Monte Carlo method, write a program in C to find the value of $\int_1^5 \frac{x}{x+\cos x}$ dx, taking sample size N = 10.

13. Write a program in C to find the largest eigenvalue in magnitude and corresponding eigenvector of the matrix $A=\begin{bmatrix} 1 & 3 & 2 \\ -1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$ using Power method.

14. Using modified Euler's method, write a program in C to evaluate $y(0.1)$ correct to two significant figures from the differential equation $\frac{dy}{dx} = y + x$, $y = 1$ when $x = 0$, taking $h = 0.05$.

15. Write a program in C to solve $\frac{dy}{dx} = xy + y^2$, using Runge-Kutta method of fourth order, given that $y(0) = 1$. Take $h = 0.2$ and find $y\ at\ x = 0.2$, 0.4, 0.6.

[Practical notebook and viva -05]